Section 8.5 Trigonometric Form of a Complex Number

**Objective:** In this lesson you learned how to multiply and divide complex numbers written in trigonometric form and how to find powers and \( n \)th roots of complex numbers.

**Important Vocabulary**

- **Real axis**
- **Imaginary axis**
- **Absolute value of a complex number** \( a + bi \)
- **\( n \)th roots of unity**

**I. The Complex Plane** (Page 637)

The complex plane is . . .

On the complex plane shown at the right, (a) label the real axis, (b) label the imaginary axis, and (c) plot and label the complex numbers \(-2 - 3i\) and \(4 + i\).

The absolute value of the complex number \( z = a + bi \) is given by

\[ |a + bi| = \sqrt{a^2 + b^2}. \]

**II. Trigonometric Form of a Complex Number** (Pages 638–639)

The **trigonometric form** of the complex number \( z = a + bi \) is

\[ z = \sqrt{a^2 + b^2} \text{cis} \tan^{-1} \left( \frac{b}{a} \right), \]

where

\[
\begin{align*}
 a &= \quad, \\
 b &= \quad, \\
 r &= \sqrt{a^2 + b^2}, \quad \text{and} \\
 \tan \theta &= \quad.
\end{align*}
\]

The number \( r \) is the \( \quad \) of \( z \), and \( \theta \) is called an \( \quad \) of \( z \).
The trigonometric form of a complex number is also called the
______________.

III. Multiplication and Division of Complex Numbers
(Pages 639–640)

Let \( z_1 = r_1(\cos \theta_1 + i \sin \theta_1) \) and \( z_2 = r_2(\cos \theta_2 + i \sin \theta_2) \) be
complex numbers. Then:

\[
\begin{align*}
z_1z_2 &= \text{______________________________} \\
\frac{z_1}{z_2} &= \text{______________________________}
\end{align*}
\]

Describe how to find the product of two complex numbers.

Describe how to find the quotient of two complex numbers.

IV. Powers of Complex Numbers (Page 641)

State DeMoivre’s Theorem.

What you should learn
How to use DeMoivre’s
Theorem to find powers
of complex numbers

V. Roots of Complex Numbers (Pages 642–644)

The complex number \( u = a + bi \) is an \textit{nth root} of the complex
number \( z \) if ____________________________.

What you should learn
How to find \( n \)th roots of
complex numbers

For a positive integer \( n \), the complex number \( z = r(\cos \theta + i \sin \theta) \)
has ____________________________ given

\[
\sqrt[n]{r}\left( \cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right), \text{ where } k = 0, 1, 2, \ldots, n - 1.
\]

Homework Assignment

Page(s)

Exercises