Section 8.3 Vectors in the Plane

Objective: In this lesson you learned how to write the component forms of vectors, perform basic vector operations, and find the direction angles of vectors.

I. Introduction (Page 615)

A directed line segment has an ____________ and a ____________.

The magnitude of the directed line segment \( \overrightarrow{PQ} \), denoted by ____________, is its ____________. The magnitude of a directed line segment can be found by . . .

II. Component Form of a Vector (Page 616)

A vector whose initial point is at the origin \((0, 0)\) can be uniquely represented by the coordinates of its terminal point \((v_1, v_2)\). This is the ________________, written \( \mathbf{v} = \langle v_1, v_2 \rangle \), where \(v_1\) and \(v_2\) are the ___________ of \(\mathbf{v}\).

The component form of the vector with initial point \(P = (p_1, p_2)\) and terminal point \(Q = (q_1, q_2)\) is

\[ \overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \mathbf{v}. \]
The **magnitude** (or length) of \( \mathbf{v} \) is:

\[
\|\mathbf{v}\| = \sqrt{\quad} = \sqrt{\quad}
\]

**Example 1:** Find the component form and magnitude of the vector \( \mathbf{v} \) that has \((1, 7)\) as its initial point and \((4, 3)\) as its terminal point.

### III. Vector Operations  (Pages 617–619)

In operations with vectors, numbers are usually referred to as ________. Geometrically, the product of a vector \( \mathbf{v} \) and a scalar \( k \) is . . .

If \( k \) is positive, \( k \mathbf{v} \) has the ________ direction as \( \mathbf{v} \), and if \( k \) is negative, \( k \mathbf{v} \) has the ________ direction.

To add two vectors geometrically, . . .

This technique is called the __________________________ for vector addition because the vector \( \mathbf{u} + \mathbf{v} \), often called the __________________________ of vector addition, is . . .

Let \( \mathbf{u} = \langle u_1, u_2 \rangle \) and \( \mathbf{v} = \langle v_1, v_2 \rangle \) be vectors and let \( k \) be a scalar (a real number). Then the sum of \( \mathbf{u} \) and \( \mathbf{v} \) is the vector:

\[
\mathbf{u} + \mathbf{v} = \langle \quad, \quad \rangle
\]

and the scalar multiple of \( k \) times \( \mathbf{u} \) is the vector:

\[
k \mathbf{u} = \langle \quad, \quad \rangle
\]

**Example 2:** Let \( \mathbf{u} = \langle 1, 6 \rangle \) and \( \mathbf{v} = \langle -4, 2 \rangle \). Find:

(a) \( 3 \mathbf{u} \)

(b) \( \mathbf{u} + \mathbf{v} \)
Let \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \) be vectors and \( c \) and \( d \) be scalars. Complete the following properties of vector addition and scalar multiplication:

1. \( \mathbf{u} + \mathbf{v} = \) __________
2. \( (\mathbf{u} + \mathbf{v}) + \mathbf{w} = \) __________
3. \( \mathbf{u} + \mathbf{0} = \) __________
4. \( \mathbf{u} + (-\mathbf{u}) = \) __________
5. \( c(d\mathbf{u}) = \) __________
6. \( (c + d)\mathbf{u} = \) __________
7. \( c(\mathbf{u} + \mathbf{v}) = \) __________
8. \( 1(\mathbf{u}) = \) __________
9. \( 0(\mathbf{u}) = \) __________
10. \( \|c\mathbf{v}\| = \) __________

**IV. Unit Vectors** (Pages 619–620)

To find a unit vector \( \mathbf{u} \) that has the same direction as a given nonzero vector \( \mathbf{v} \), . . .

In this case, the vector \( \mathbf{u} \) is called a __________

**Example 3:** Find a unit vector in the direction of \( \mathbf{v} = \langle -8, 6 \rangle \).

Let \( \mathbf{v} = \langle v_1, v_2 \rangle \). Then the standard unit vectors can be used to represent \( \mathbf{v} \) as \( \mathbf{v} = \) __________, where the scalar \( v_1 \) is called the __________ and the scalar \( v_2 \) is called the __________. The vector sum \( v_1\mathbf{i} + v_2\mathbf{j} \) is called a __________ of the vectors \( \mathbf{i} \) and \( \mathbf{j} \).

**Example 4:** Let \( \mathbf{v} = \langle -5, 3 \rangle \). Write \( \mathbf{v} \) as a linear combination of the standard unit vectors \( \mathbf{i} \) and \( \mathbf{j} \).

**Example 5:** Let \( \mathbf{v} = 3\mathbf{i} - 4\mathbf{j} \) and \( \mathbf{w} = 2\mathbf{i} + 9\mathbf{j} \). Find \( \mathbf{v} + \mathbf{w} \).
V. Direction Angles (Page 621)

If \( \mathbf{u} \) is a unit vector and \( \theta \) is its direction angle, the terminal point of \( \mathbf{u} \) lies on the unit circle and
\[
\mathbf{u} = \langle x, y \rangle = \text{______________} = \text{______________}
\]

Now, if \( \mathbf{v} \) is any vector that makes an angle \( \theta \) with the positive \( x \)-axis, it has the same direction as \( \mathbf{u} \) and
\[
\mathbf{v} = \text{______________} = \text{______________}
\]

If \( \mathbf{v} \) can be written as \( \mathbf{v} = ai + bj \), then the direction angle \( \theta \) for \( \mathbf{v} \) can be determined from \( \tan \theta = \text{__________} \).

Example 6: Let \( \mathbf{v} = -4i + 5j \). Find the direction angle for \( \mathbf{v} \).

VI. Applications of Vectors (Pages 622–623)

Describe several real-life applications of vectors.

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**What you should learn**

- How to find the direction angles of vectors
- How to use vectors to model and solve real-life problems

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**Homework Assignment**

Page(s)

Exercises