Section 5.2 Logarithmic Functions and Their Graphs

Objective: In this lesson you learned how to recognize, evaluate, and graph logarithmic functions.

I. Logarithmic Functions (Pages 401–402)

The logarithmic function with base \( a \) is defined as

\[ \log_a x \] , for \( x > 0 \) and \( 0 < a \neq 1 \), if and only if \( x = a^y \).

The logarithmic function with base \( a \) is the ____________ of the exponential function \( f(x) = a^x \).

The equation \( x = a^y \) in exponential form is equivalent to the equation ____________ in logarithmic form.

When evaluating logarithms, remember that a logarithm is a(n) ______________. This means that \( \log_a x \) is the ______________ to which \( a \) must be raised to obtain ________.

Example 1: Use the definition of logarithmic function to evaluate \( \log_5 125 \).

Example 2: Use a calculator to evaluate \( \log_{10} 300 \).

Complete the following properties of logarithms:

1) \( \log_a 1 = \) _________
2) \( \log_a a = \) _________
3) \( \log_a a^x = \) _________ and \( a^{\log_a x} = \) _________
4) If \( \log_a x = \log_a y \), then ____________.
Example 3: Solve the equation $\log_7 x = 1$ for $x$.

II. Graphs of Logarithmic Functions (Pages 403–404)

For $a > 1$, is the graph of $y = \log_a x$ increasing or decreasing over its domain? ________________

For the graph of $y = \log_a x$, $a > 1$, the domain is ________________, the range is ________________, and the intercept is ________________.

Also, the graph has ________________ as a vertical asymptote. The graph of $y = \log_a x$ is a reflection of the graph of $y = a^x$ about ________________.

Example 4: Sketch the graph of the function $f(x) = \log_3 x$.

III. The Natural Logarithmic Function (Pages 405–406)

Complete the following properties of natural logarithms:

1) $\ln 1 = \underline{\phantom{0}}$
2) $\ln e = \underline{\phantom{0}}$
3) $\ln e^x = \underline{\phantom{0}}$ and $e^{\ln x} = \underline{\phantom{0}}$
4) If $\ln x = \ln y$, then ________________.

Example 5: Use a calculator to evaluate $\ln 10$.
Example 6:  Find the domain of the function  \( f(x) = \ln(x + 3) \).

IV. Applications of Logarithmic Functions  (Page 407)
Describe a real-life situation in which logarithms are used.

Example 7:  A principal \( P \), invested at 6% interest and compounded continuously, increases to an amount \( K \) times the original principal after \( t \) years, where \( t \) is given by \( t = \frac{\ln K}{0.06} \). How long will it take the original investment to double in value? To triple in value?

What you should learn
How to use logarithmic functions to model and solve real-life applications

Additional notes

Homework Assignment
Page(s)
Exercises